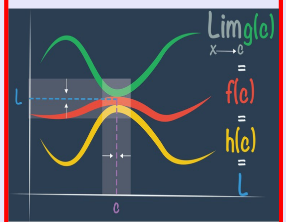
Math 261

Spring 2021

Lecture 25



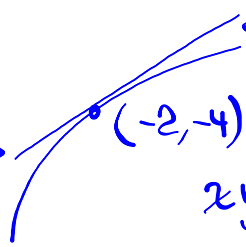
Class QZ 10

Find eqn of the tan. line to the graph of $xy = 8$ at the point with $x = -2$.

$xy = 8$

$(-2)y = 8$

$y = -4$



$(-2,-4)$

$m = \left.\dfrac{dy}{dx}\right|_{(-2,-4)}$

$= \dfrac{-8}{(-2)^2} \Rightarrow \boxed{m = -2}$

$xy = 8$

$y = \dfrac{8}{x} \qquad \dfrac{dy}{dx} = \dfrac{-8}{x^2}$

$y - y_1 = m(x - x_1)$

$y - -4 = -2(x - -2) \qquad y + 4 = -2x - 4 \qquad \boxed{y = -2x - 8}$

March 24, 2021

Find $\frac{dy}{dx}$ for $x^2 = \frac{\cot y}{1+\csc y}$

$x^2(1+\csc y) = \cot y$

$x^2 + x^2\csc y = \cot y$

$$\frac{d}{dx}\left[x^2\right] + \frac{d}{dx}\left[x^2\csc y\right] = \frac{d}{dx}\left[\cot y\right]$$

$$2x + 2x\cdot\csc y + x^2\cdot -\csc y\cot y\cdot\frac{dy}{dx} = -\csc^2 y\cdot\frac{dy}{dx}$$

$$2x + 2x\csc y = x^2\csc y\cot y\boxed{\frac{dy}{dx}} - \csc^2 y\boxed{\frac{dy}{dx}}$$

$$2x + 2x\csc y = \left(x^2\csc y\cot y - \csc^2 y\right)\frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = \frac{2x + 2x\csc y}{x^2\csc y\cot y - \csc^2 y}}$$

use Your calc .4048 24...

Use linear approximation to estimate $\frac{1.96}{1.96^2+1}$

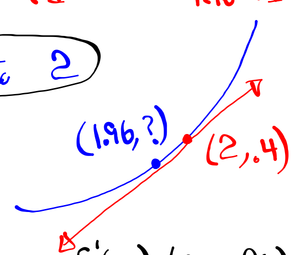
Let $f(x) = \frac{x}{x^2+1}$, $\frac{1.96}{1.96^2+1} = f(1.96)$

$F(2) = \frac{2}{2^2+1} = \frac{2}{5}$

$= .4$

1.96 is close to 2

$x_0 = 2$



$f(x) = \frac{x}{x^2+1}$

$F'(x) = \frac{1(x^2+1) - x\cdot 2x}{(x^2+1)^2}$

$= \frac{1-x^2}{(x^2+1)^2}$

$f'(2) = \frac{1-4}{5^2}$

$$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$$

$$= f(2) + f'(2)(x-2)$$

$$= .4 + \frac{-3}{25}(x-2)$$

$$f(1.96) \approx .4 - \frac{3}{25}(1.96-2) = .4 - \frac{3}{25}(-.04) = .4 + \frac{.12\cdot 4}{25\cdot 4}$$

$= .4 + .0048$

$= \boxed{.4048}$

Use linear approximation to evaluate

$\sin 46^\circ + \cos 46^\circ$

Let $f(x) = \sin x + \cos x$ Near 45°

$x_0 = 45^\circ$

$f(x) \approx f(x_0) + f'(x_0)(x - x_0)$

1) $f(45^\circ) = \sin 45^\circ + \cos 45^\circ = \frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} = \sqrt{2}$

2) $f'(x) = \cos x - \sin x$

3) $f'(45^\circ) = \cos 45^\circ - \sin 45^\circ = 0$

$f(x) \approx f(45^\circ) = \sqrt{2}$

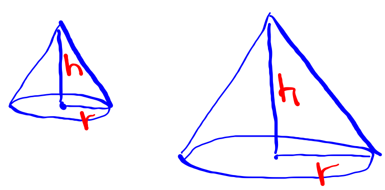
$\underbrace{\sin 46^\circ + \cos 46^\circ}_{\text{by Calc}} \approx \underbrace{\sqrt{2}}_{\text{by Calc.}}$

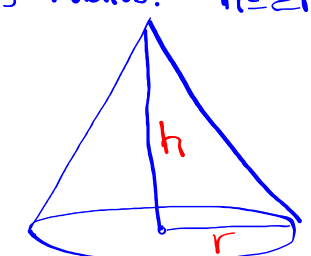
1.414 1.414

Grain is pouring at the rate of 8 ft³/min.

It forms Conical Pile Such that its $\frac{dV}{dt} = 8\,ft^3/min$

its altitude is twice its radius. $h = 2r$





How fast is its altitude changing when the Pile is 6 ft high?

$\frac{dh}{dt} = ?$ when $h = 6$

$h = 2r \rightarrow r = \frac{h}{2}$

$V = \frac{1}{3}\pi r^2 h$ $V = \frac{1}{3}\pi \left(\frac{h}{2}\right)^2 \cdot h$ $V = \frac{\pi}{12} h^3$

$\frac{dV}{dt} = \frac{\pi}{12} \cdot 3h^2 \cdot \frac{dh}{dt}$ $8 = \frac{\pi}{4}(6)^2 \frac{dh}{dt}$ $\boxed{\frac{dh}{dt} = \quad ft/min}$

An object is moving along the path given by $y = \sqrt{x^3+17}$

when the object is at the point $(2,5)$, how fast x is changing if y is increasing at the rate of 2 units/sec?

$\frac{dx}{dt}$ at $(2,5)$ and $\frac{dy}{dt} = 2$ units/sec.

$y = \sqrt{x^3+17}$

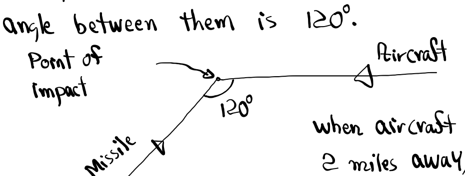
$y^2 = x^3+17$

$2y\frac{dy}{dt} = 3x^2\frac{dx}{dt}$

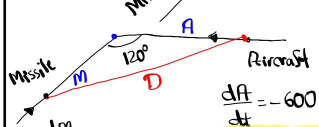
$2\cdot 5\cdot 2 = 3\cdot 2^2\cdot\frac{dx}{dt}$

$\boxed{\frac{dx}{dt} = \quad \text{units/sec.}}$

An aircraft is flying at a constant altitude with constant speed of 600 mph. An antiaircraft missile is fired on a straight line to the flight path. Assume the angle between them is 120°.



When aircraft is 2 miles away, and missile is 4 miles away, missile is flying at 1200 mph. How fast is the distance between them decrease?



$\frac{dA}{dt} = -600$

$\frac{dM}{dt} = -1200$

Law of Cosines

$D^2 = A^2 + M^2 - 2AM\cos 120°$

$2D\frac{dD}{dt} = 2A\cdot\frac{dA}{dt} + 2M\frac{dM}{dt} + \frac{dA}{dt}\cdot M + A\cdot\frac{dM}{dt}$

$-2\cos 120° = -2\left(-\frac{1}{2}\right) = 1$

$2\cdot\sqrt{28}\cdot\frac{dD}{dt} = 2\cdot 2\cdot(-600) + 2(4)\cdot(-1200) + (-600)\cdot 4 + 2\cdot(-600)$

$\boxed{\frac{dD}{dt} = -600\sqrt{7}\text{ mph}}$

Check this Answer

$D^2 = A^2 + M^2 - 2AM\cos 120°$
$= 2^2 + 4^2 - 2(2)(4)\cdot -\frac{1}{2}$
$= 4 + 16 + 8 = 28$

$D = \sqrt{28}$

$f(x) = 3x^{5/3} - 15x^{2/3}$

1) Find $f'(x)$ $f'(x) = \cancel{3} \cdot \frac{5}{\cancel{3}} x^{\frac{5}{3}-1} - \overset{5}{\cancel{15}} \cdot \frac{2}{\cancel{3}} x^{\frac{2}{3}-1}$

$f'(x) = 5x^{2/3} - 10x^{-1/3}$

2) write $f'(x)$ in factored form.

$f'(x) = 5x^{-1/3}(x^1 - 2)$ $f'(x) = \frac{5(x-2)}{\sqrt[3]{x}}$

3) Find x-values where $f'(x)$ is zero or undefined.

$F'(x) = 0 \rightarrow x - 2 = 0$ $x = 2$

$f'(x)$ undefined $\sqrt[3]{x} = 0$ $x = 0$

4) Find points on the graph of $f(x)$ where $f'(x)$ is zero or undefined.

$f(0) = 3(0)^{5/3} - 15(0)^{2/3} = \boxed{0}$ $(0, 0)$

$f(2) = 3(2)^{5/3} - 15(2)^{2/3} = \boxed{?}$ $(2, ?)$